SOLUCIONARIO DEL SIMULACRO PRESENCIAL DE MATEMÁTICA
REPASO UNI - 2020

Aritmética

Resolución N.°1

$A=\{x\in N/2x\leq 13\}$

$2x\leq 13\rightarrow x\leq 6,5$

$x=1, 2, 3, 4, 5, 6$

$\downarrow$

$\in\mathbb{N}$ $\boxed{A=\{1; 2; 3; 4; 5; 6\}}$

$B=\{x\in A\ /\ (x^2-2x)\notin A\}$

$x=1; 2; \cancel{3}; 4; 5; 6$

pero no puede ser:

$x^2-2x=\underbrace{x(x-2)}_{\notin A}=-1; 0; \underset{\in A}{3}; 8; 15; 24$

$\rightarrow \boxed{B=\{1; 2; 4; 5; 6\}}$

Necesitamos: $A-B=\{3\}$

I. $\exists\, x\in A\ /\ x^2-5>4.$

$\quad\ \ 4 \qquad x^2>9$

$\qquad\qquad 16>9$ (V)

II. $\forall\, x\in (A-B)\ /\ 2x+5<8.$

$\quad\ \ 3 \qquad 2(3)+5<8$

$\qquad\qquad 11<8$ (F)

III. $\exists\, x\in (A-B)\ /\ x^2\in B.$

$\quad\ \ 3 \qquad 3^2=9\notin B$ (F)

$\therefore$ VFF

Clave: D

Resolución N.°2

$$\overline{a(a+1)b(b-2)c(c/2)(\sqrt{d})}$$

✓ $a+1<10$ → $\boxed{a<9}$

✓ $b-2\geq 2$ → $\boxed{b\geq 2}$

$c=$par

a	b	c	$\sqrt{d}$
1	2		0
2	3		1
3	4		2
4	5	0	3
5	6	2	⋮
6	7	4	
7	8	6	
8	9	8	9
8	× 8	× 5	× 10

$\therefore$ 3200 números

Clave: C

Resolución N.°3

Nos piden

$\dfrac{2dk^2}{2dk}=\dfrac{dk}{d}=k\in\mathbb{Z}^+$

Dato:

$2dk^2-d=136$

$d(2k^2-1)=136=8\times 17$

$\downarrow\qquad\downarrow$

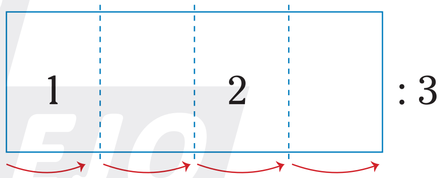
$8\quad \underbrace{3}_{17}\qquad d=8; k=3$

Piden: Suma antecedentes: $2(8)(3^2)+8(3)$

$\therefore$ Suma antecedente=168

Clave: B

Resolución N.°4



18 obr.	$(18+x)$	$18+2x$	$18+3x$
8 días	1 día	1 día	1 día

$18\times 8=\dfrac{(18+x)1+(18+2x)1+(18+3x)1}{2}$

$\therefore x=39$

Clave: C

Resolución N.°5

Sean las dificultades:

Pantalón: P

Camisa: C

* $\overset{4}{\cancel{12}}\cdot P\cdot 6=\overset{5}{\cancel{15}}\cdot C\cdot \cancel{4}\rightarrow \dfrac{P}{C}=\dfrac{5}{6}$

* Detergente (gr)	Pantalón y/o camisas	Días
50	12 P o 15 C	6 días y 4 días
x	3 P y 4 C	15 días

$\dfrac{\text{Detergente}}{(\text{pantalón y camisas})(\text{días})}=\text{cte}$

$\dfrac{50}{12\times P\times 6}=\dfrac{x}{(3P+4C)15}$

$\dfrac{\overset{25}{\cancel{50}}}{\underset{4}{\cancel{12}}\times\cancel{5}\times\underset{2}{\cancel{6}}}=\dfrac{x}{\underbrace{(3\times 5+4\times 6)}_{\underset{13}{\cancel{39}}}\underset{3}{\cancel{15}}}$

$\therefore x=81,25$ gr

Clave: A

Resolución N.°6

Sea la variable aleatoria x: nº lanzamientos

* $x=1 \rightarrow P(x=1)=\frac{1}{2}$

* $x=2 \rightarrow P(x=2)=\frac{2}{4}=\frac{1}{2}$

* $E(x)=\Sigma x_i \times P(x_i)$

$E(x)=1\times\frac{1}{2}+2\times\frac{1}{2}=1{,}5$

Clave: B

Resolución N.°7

$1492^{\overline{abc}}=\overset{o}{40}+24$

$(\overset{o}{40}+12)^{\overline{abc}}=\overset{o}{40}+24$

$12^{\overline{abc}}=\overset{o}{40}+24$

* $12^2=\overset{o}{40}+24$

$\rightarrow 12^{\overset{o}{2}}=\overset{o}{40}+24$

$\Rightarrow \overline{abc}=\overset{o}{2}$

Piden máximo $\overline{abc}=998$

Clave: E

Resolución N.°8

$$\frac{(\text{nº oscilaciones})(\text{longitud})}{(\text{masa})(\text{energía})^2}=\text{cte}$$

Nº oscilaciones	12	7
Longitud	5	4
Masa	x	$x+20$
Energía	5	3

$$\frac{\overset{3}{\cancel{12}}\times\cancel{5}}{x\times\underset{5}{\cancel{25}}}=\frac{7\times\cancel{4}}{(x+20)9}$$

$27(x+20)=35x$

$x=67{,}5$

Piden: $(x+20)=87{,}5$

Clave: A

Resolución N.°9

$N=\left(\overline{mnpq}_7\right)^{\overline{mnp}_4}$

$N=\overline{mnpq}_7^{\,(m\times4^2+n\times4+p)}$

$N=(\overline{mnpq}_7)^{16m}\times(\overline{mnpq}_7)^{4n}\times(\overline{mnpq}_7)^{p}$

$N=(\overline{mnpq}_7{}^{m})^{16}(\overline{mnpq}_7{}^{n})^{4}\times\overline{mnpq}_7{}^{p}$

$N=(\overset{o}{11}+5)^{16}(\overset{o}{11}-4)^{4}(\overset{o}{11}+9)$ $\quad$ $\boxed{5^5=\overset{o}{11}+1}$

$N=(\overset{o}{11}+5^{16})(\overset{o}{11}+4^{4})(\overset{o}{11}+9)$

$N=(\overset{o}{11}+(5^5)^3\times5^1)(\overset{o}{11}+3)(\overset{o}{11}+9)$

$N=(\overset{o}{11}+(\overset{o}{11}+1)\times5)(\overset{o}{11}+5)$

$N=(\overset{o}{11}+5)(\overset{o}{11}+5)=\overset{o}{11}+25$

$N=\overset{o}{11}+3$

$\therefore$ Residuo=3

Clave: B

Resolución N.°10

$\underbrace{\frac{N}{D}}=0{,}1\widehat{abcd}$; Σ cifras de "N" es máxima

propia e irreductible

$D<345$

$\downarrow$

101×2 o $\cancel{101\times5}$

$\frac{N}{202}\rightarrow\{21; 23; 25; 27; 29; 31; \ldots; 39\}$ $\quad$ Σcifras=12(máx)

$\rightarrow\frac{39}{202}=0{,}1\widehat{9306}$

Piden: Suma de cifras de la parte periódica$=9+3+0+6=18$

Clave: C

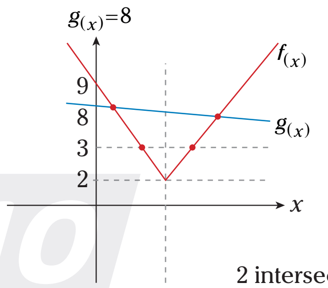
Álgebra

Resolución N.°11

$|x-2|+|x-3|+|x-4|=8$

Piden el número de soluciones reales; que será lo mismo que el número de intersecciones.

De $f_{(x)}=|x-2|+|x-3|+|x-4|$



2 intersecciones

$\therefore$ 2 soluciones

Clave: C

Resolución N.°12

Indique V o F

I. Falso

$a_n=\{n\}$ es monótona creciente pero diverge

II. Verdadero

$b_n=\{(-1)^n\}$ es acotada

$|b_n|\leq1\ \forall n$ pero no converge

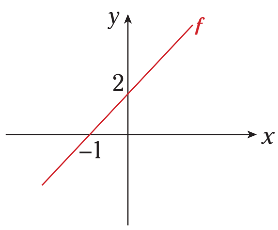
III. Verdadero

$c_n=\{-n\}$ decreciente pero no converge

Clave: D

Resolución N.°13

Para: f



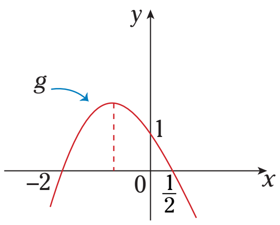
$\frac{x}{-1}+\frac{y}{2}=1$

$-2x+y=2$

$y=2x+2$

$\boxed{f_{(x)}=2x+2}$

Para: g



$$g_{(x)}=a(x+2)\left(x-\frac{1}{2}\right)$$

$$g_{(0)}=a(2)\left(-\frac{1}{2}\right)=1$$

$$\rightarrow a=-1$$

$$\boxed{g_{(x)}=-x^2-\frac{3}{2}x+1}$$

$$\therefore f+g=-x^2-\frac{3}{2}x+1+2x+2=-x^2+0{,}5x+3;\ \forall x\in\mathbb{R}$$

Clave: C

Resolución N.°14

Por inducción

$$A^2=\begin{pmatrix}1&0&1\\0&1&0\\0&0&1\end{pmatrix}\begin{pmatrix}1&0&1\\0&1&0\\0&0&1\end{pmatrix}=\begin{pmatrix}1&0&2\\0&1&0\\0&0&1\end{pmatrix}$$

$$A^3=\begin{pmatrix}1&0&2\\0&1&0\\0&0&1\end{pmatrix}\begin{pmatrix}1&0&1\\0&1&0\\0&0&1\end{pmatrix}=\begin{pmatrix}1&0&3\\0&1&0\\0&0&1\end{pmatrix}$$

$\vdots$

$$A^n=\begin{pmatrix}1&0&n\\0&1&0\\0&0&1\end{pmatrix}$$

$\therefore \sum \text{elementos}: n+3$

Clave: E

Resolución N.°15

Fijando "n"

$$S=\text{sen}1-2\text{sen}\frac{1}{2}+2\text{sen}\frac{1}{2}-3\text{sen}\frac{1}{3}+...+(n-1)\text{sen}\left(\frac{1}{n-1}\right)-n\,\text{sen}\frac{1}{n}$$

$$S=\text{sen}1-n\,\text{sen}\frac{1}{n}$$

$$\lim_{n\to\infty} S=\text{sen}1-\underbrace{\lim_{n\to\infty}\frac{\text{sen}\left(\frac{1}{n}\right)}{\frac{1}{n}}}_{\text{uno}}$$

$$S=\text{sen}1-1$$

Clave: C

Resolución N.°16

$$f_{(x)}=x^2+\sqrt{(x-2)^2}=x^2+|x-2|$$

$$f_{(x)}=\begin{cases}x^2+x-2; & x\geq 2\\ x^2-x+2; & x<2\end{cases}$$

$$g_{(x)}=\frac{3x+11}{2}$$

$f\cap n$:

I) $x^2+x-2=\dfrac{3x+11}{2}$ para $x\geq 2$

$\rightarrow x=3 \vee x=-5/2$

$\therefore x=3$

II) $x^2-x+2=\dfrac{3x+11}{2}$ para $x<2$

$\rightarrow x=7/2 \ \vee \ x=-1$

$\therefore x=-1$

Los pares:

(3; 10) y (–1; 4)

Luego: $3+10+4-1=16$

Clave: B

Resolución N.°17

$S=a_1+a_2+a_3+...$

$S=a_1+a_1q+a_1q^2+...=a_1(1+q+q^2+...)$

$$S=\frac{a_1}{1-q}\ \longleftrightarrow\ q=\frac{S-a_1}{S}\ ....\ \alpha$$

Piden:

$A=-a_1+a_2-a_3+a_4-a_5+a_6\ ...$

$A=-a_1+a_1q-a_1q^2+a_1q^3-a_1q^4+a_1q^5-\ ...$ (+)

$qA=-a_1q+a_1q^2-a_1q^3+a_1q^4-a_1q^5+\ ...$

$A(1+q)=-a_1$ reemplaza α

$$A\left(1+\frac{S-a_1}{S}\right)=-a_1$$

$$A\left(\frac{2S-a_1}{S}\right)=-a_1$$

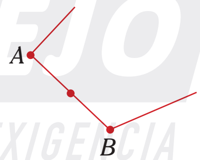
$$\therefore A=\frac{-a_1S}{2S-a_1}$$

Clave: D

Resolución N.°18

I. (F) El mínimo (si existe) se encuentra en un vértice (punto extremo)

II. (V)



Si $f_{(A)}=f_{(B)}=\text{mín}\,f \leftrightarrow$

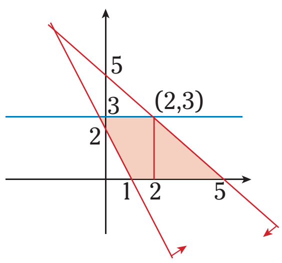
$\overline{AB}$ contiene las infinitas soluciones óptimas

III. (V) Cada punto extremo es determinado por la intersección de las rectas como un P.P.L. tiene finitas restricciones lineales por lo que genera finitos puntos extremos.

Clave: E

Resolución N.°19

De las restricciones



Pendiente de rectas de nivel

$$\frac{y}{x}=-1{,}5=-\frac{3}{2}$$

Luego: $f_{(x;\,y)}=2y+3x$

$f_{(2;\,3)}=2(3)+3(2)=12$

$f_{(5;\,0)}=3(5)+2(0)=15$

$f_{(1;\,0)}=3(1)+2(0)=3$

$f_{(0;\,3)}=3(0)+2(3)=6$

$\therefore$ máx $f_{(x;\,y)}=15$

Clave: E

Resolución N.°20

$2^{y-x}=2^{-8} \rightarrow \boxed{x-y=8}$

$\log y+\log x=\left(\dfrac{2^{10}}{2^{3}}\right)$

$\log(xy)=\log(2^7)$

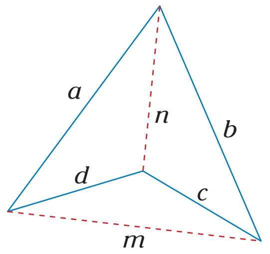
$\therefore\ xy=2^7=128$

Clave: B

Geometría

Resolución N.°21

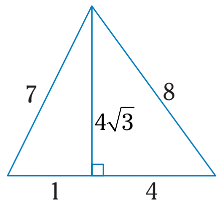
I. (F)

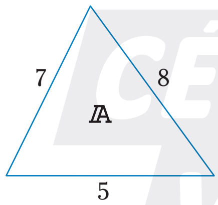


$$\begin{aligned} n&<a+d \quad + \\ n&<b+c \\ m&<a+b \\ m&<d+c \\ \hline 2m+2n&<2a+2b+2c+2d \\ m+n&<a+b+c+d \end{aligned}$$

II. (F)

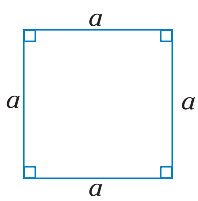
Si recordamos

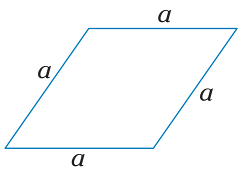




$\mathbb{A}=\dfrac{5(4\sqrt{3})}{2}=10\sqrt{3}$

III. (F)

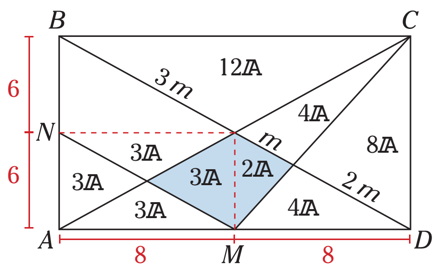




Clave: E

Resolución N.°22

Piden $\mathbb{A}_{\text{somb.}}$



$48\mathbb{A}=12(16)$

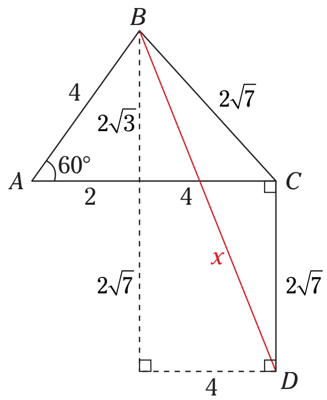
$\rightarrow \mathbb{A}=4$

$\therefore\ \mathbb{A}_{\text{somb.}}=5\mathbb{A}=5(4)=20$

Clave: E

Resolución N.°23

Piden $(BD)^2$



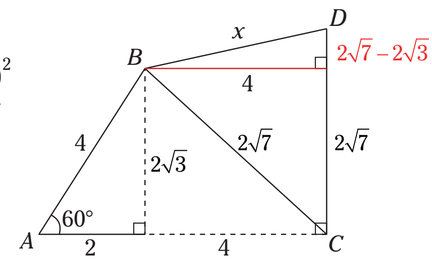
Por T. de Pitágoras

$x^2=(4)^2+(2\sqrt{3}+2\sqrt{7})^2$

$x^2=16+12+8\sqrt{21}+28$

$x^2=56+8\sqrt{21}$

$x^2=8(7+\sqrt{21})$



$x^2=(4)^2+(2\sqrt{7}-2\sqrt{3})^2$

$x^2=16+28+12-8\sqrt{21}$

$x^2=56-8\sqrt{21}$

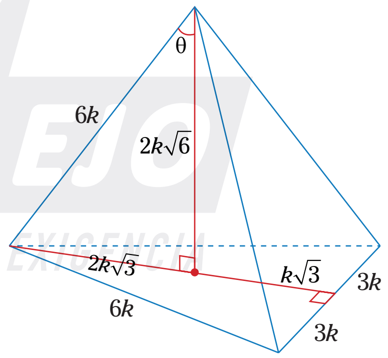
$x^2=8(7-\sqrt{21})$

Debe indicar D ubicado en la región exterior relativa a $\overline{BC}$.

Clave: E

Resolución N.°24

Piden $\cos\theta$



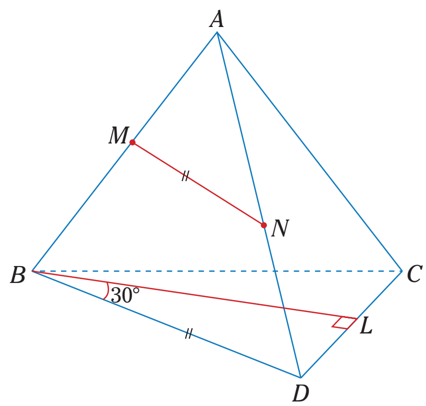
$\cos\theta=\dfrac{2k\sqrt{6}}{6k}=\dfrac{\sqrt{6}}{3}=\dfrac{\sqrt{2}\sqrt{3}}{\sqrt{3}\sqrt{3}}$

$\cos\theta=\dfrac{\sqrt{2}}{\sqrt{3}}$

Clave: E

Resolución N.°25

Piden m∢$(\overline{MN};\ \overline{BL})$



Se observa: $\overline{MN}//\overline{BD}$

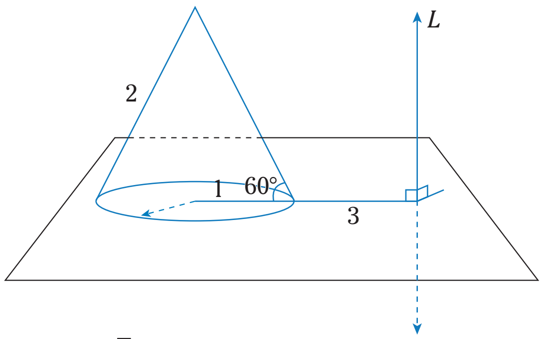
m∢$(\overline{MN};\ \overline{BL})$=m∢$DBL$

m∢$(\overline{MN};\ \overline{BL})=30°$

Clave: D

Resolución N.°26

Piden $\mathbb{V}_{SG}$



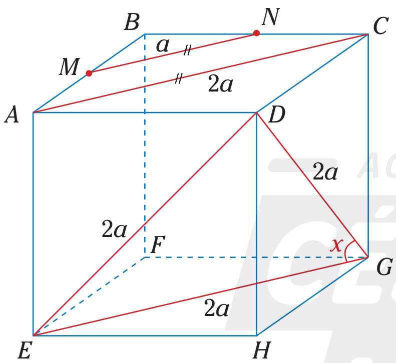
$$\mathbb{V}_{SG}=2\pi(4)\frac{2^2\sqrt{3}}{4}$$

$$\mathbb{V}_{SG}==8\pi\sqrt{3}$$

Clave: C

Resolución N.°27

Piden m∢($\overline{MN}$; $\overline{DG}$)



ΔEDG: equilátero

x=60°

Clave: B

Resolución N.°28

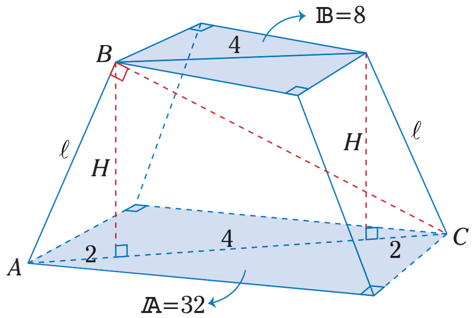
Piden: Volumen$_{\text{(tronco de pirámide)}}$

Datos:

Área de la base menor: $\mathbb{B}$=8

Área de la base mayor: $\mathbb{A}$=32

Graficamos:



⊿ABC: $(H)^2$=(2)(6)

H=$2\sqrt{3}$

Aplicamos fórmula de volumen de tronco de pirámide:

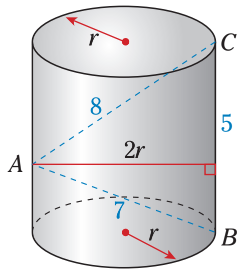
$$\text{Volumen}_{\left(\substack{\text{tronco de}\\ \text{pirámide}}\right)}=\frac{2\sqrt{3}}{3}\left[32+8+\sqrt{(32)(8)}\right]$$

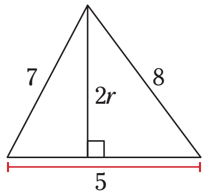
$$\therefore \text{Volumen}_{\left(\substack{\text{tronco de}\\ \text{pirámide}}\right)}=\frac{112\sqrt{3}}{3}$$

Clave: D

Resolución N.°29

Piden: V_{cilindro}





$$2r=\frac{2}{5}\sqrt{10(5)(2)(3)}$$

$$2r=\frac{2}{5}(10\sqrt{3})=4\sqrt{3}$$

$$r=2\sqrt{3}$$

$$V_c=\pi(2\sqrt{3})^2(5)\rightarrow V_c=60\pi$$

Clave: E

Resolución N.°30

Piden: $V_{\text{(tronco de cono)}}$

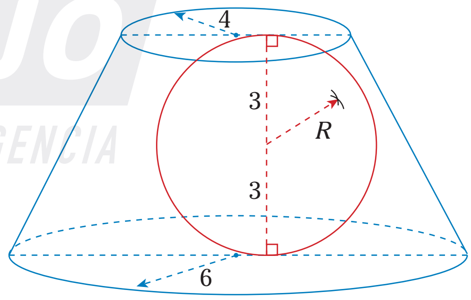
Dato:

$V_{\text{(esfera)}}=36\pi$

$$\frac{4\pi}{3}(R)^3=36\pi$$

$$R=3$$

Graficamos:



Aplicamos fórmula de volumen de tronco de cono:

$$V_{\left(\substack{\text{tronco}\\ \text{de cono}}\right)}=\frac{\pi(6)}{3}\left[6^2+4^2+(6)(4)\right]$$

$$\therefore V_{\left(\substack{\text{tronco}\\ \text{de cono}}\right)}=152\pi$$

Clave: B

Trigonometría

Resolución N.°31

$\tan^4 x$=?

Condición

$$\cos 2x=\frac{\operatorname{sen}40^\circ}{1+\cos 40^\circ}$$

$$\cos 2x=\frac{\not{2}\operatorname{sen}20^\circ\,\not{\cos 20^\circ}}{\not{2}\cos^2 20^\circ}$$

$$\cos 2x=\tan 20^\circ$$

$$\frac{1-\tan^2 x}{1+\tan^2 x}=\frac{\tan 20^\circ}{1}$$

$$\rightarrow \tan^2 x=\frac{1-\tan 20^\circ}{1+\tan 20^\circ}$$

a senos y cosenos

$$\tan^2 x=\frac{\cos 20^\circ-\operatorname{sen}20^\circ}{\cos 20^\circ+\operatorname{sen}20^\circ}$$

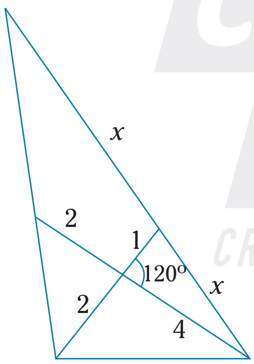
al cuadrado:

$$\tan^4 x=\frac{(\cos 20^\circ-\operatorname{sen}20^\circ)^2}{(\cos 20^\circ+\operatorname{sen}20^\circ)^2}$$

$$\therefore \tan^4 x=\frac{1-\operatorname{sen}40^\circ}{1+\operatorname{sen}40^\circ}$$

Clave: A

Resolución N.°32



El mayor: $2x$

Por teorema de cosenos

$$x^2=4^2+1^2-2(4)(1)\cos 120^\circ$$

$$x^2=21\rightarrow x=\sqrt{21}$$

$\therefore$ La longitud del mayor lado es $2\sqrt{21}$

Clave: B

Resolución N.°33

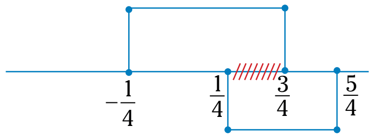
Máximo valor de A siendo:

$$A=\operatorname{arcsen}\sqrt{x+\frac{1}{4}}+\operatorname{arcsen}\sqrt{x-\frac{1}{4}}$$

A está definida si:

$$0\leq x+\frac{1}{4}\leq 1 \quad \wedge \quad 0\leq x-\frac{1}{4}\leq 1$$

$$-\frac{1}{4}\leq x\leq\frac{3}{4} \quad \wedge \quad \frac{1}{4}\leq x\leq\frac{5}{4}$$



De las relaciones

$$\frac{1}{4}\leq x\leq\frac{3}{4}$$

Como arcsen es creciente

$$\rightarrow A_{\text{máx}}=\operatorname{arcsen}\sqrt{\frac{3}{4}+\frac{1}{4}}+\operatorname{arcsen}\sqrt{\frac{3}{4}-\frac{1}{4}}$$

$$\therefore A_{\text{máx}}=\frac{\pi}{2}+\frac{\pi}{4}=\frac{3\pi}{4}$$

Clave: B

Resolución N.°34

$$\arctan(-2x)\geq\frac{1}{4}\operatorname{arccot}(2x)$$

$$-\arctan 2x\geq\frac{1}{4}\left(\frac{\pi}{2}-\arctan 2x\right)$$

$$-\arctan 2x\geq\frac{\pi}{8}-\frac{1}{4}\arctan 2x$$

$$-\frac{3}{4}\arctan 2x\geq\frac{\pi}{8}\rightarrow\arctan 2x\leq-\frac{\pi}{6}$$

Como tan es creciente

$$\rightarrow 2x\leq-\frac{\sqrt{3}}{3}\rightarrow x\leq-\frac{\sqrt{3}}{6}$$

$$\equiv x\in\left\langle-\infty;-\frac{\sqrt{3}}{6}\right]$$

Clave: A

Resolución N.°35

Ran$f$=?

$$f_{(x)}=2(\cos 2x-3)(-2-\operatorname{sen}^2 x);\, x\in\mathbb{R}$$

$$f_{(x)}=2(1-2\operatorname{sen}^2 x-3)(-2-\operatorname{sen}^2 x)$$

$$f_{(x)}=4(\operatorname{sen}^2+1)(\operatorname{sen}^2 x+2)$$

$$f_{(x)}=4(\operatorname{sen}^4 x+3\operatorname{sen}^2 x+2)$$

$$f_{(x)}=4\left[\left(\operatorname{sen}^2 x+\frac{3}{2}\right)^2-\frac{1}{4}\right]$$

$$x\in\mathbb{R}\rightarrow 0\leq\operatorname{sen}^2 x\leq 1\rightarrow\frac{3}{2}\leq\operatorname{sen}^2 x+\frac{3}{2}\leq\frac{5}{2}$$

$$\frac{9}{4}\leq\left(\operatorname{sen}^2 x+\frac{3}{2}\right)^2\leq\frac{25}{4}$$

$$2\leq\left(\operatorname{sen}^2 x+\frac{3}{2}\right)^2-\frac{1}{4}\leq 6$$

$$8\leq\underbrace{4\left[\left(\operatorname{sen}^2 x+\frac{3}{2}\right)^2-\frac{1}{4}\right]}_{f_{(x)}}\leq 24$$

$$\therefore \operatorname{Ran}f=[8;24]$$

Clave: C

Resolución N.°36

$$\tan\left(2\arccos\left(-\frac{\sqrt{11}}{6}\right)\right)$$

$\arccos(-x)=\pi-\arccos x$

$$\tan\left[2\left(\pi-\arccos\left(\frac{\sqrt{11}}{6}\right)\right)\right]$$

$$\tan\left[2\pi-2\arccos\left(\frac{\sqrt{11}}{6}\right)\right]$$

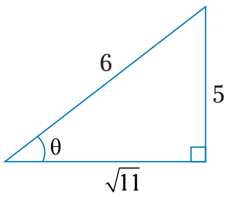
$$-\tan\left(2\arccos\left(\frac{\sqrt{11}}{6}\right)\right)$$

Sea $\arccos\frac{\sqrt{11}}{6}=\theta\rightarrow\cos\theta=\frac{\sqrt{11}}{6}$

Reemplazando:

$$-\tan(2\theta)=-\frac{2\tan\theta}{1-\tan^2\theta}\ \ldots\ (1)$$

De $\cos\theta=\frac{\sqrt{11}}{6}$



$$\tan\theta=\frac{5}{\sqrt{11}}$$

Reemplazando en (1)

$$\text{Lo pedido}=\frac{-2\left(\frac{5}{\sqrt{11}}\right)}{1-\left(\frac{5}{\sqrt{11}}\right)^2}=\frac{5\sqrt{11}}{7}$$

Clave: A

Resolución N.°37

Soluciones enteras de:

$\cos^2 2x+\cos^2 x\leq 1;\quad 0<x<2\pi$

$2\cos^2 2x+2\cos^2 x\leq 2$

$2\cos^2 2x+1+\cos 2x\leq 2$

$2\cos^2 2x+\cos 2x-1\leq 0$

$2\cos 2x \qquad -1$

$\cos 2x \qquad +1$

$(2\cos 2x-1)\underbrace{(\cos 2x+1)}\leq 0$

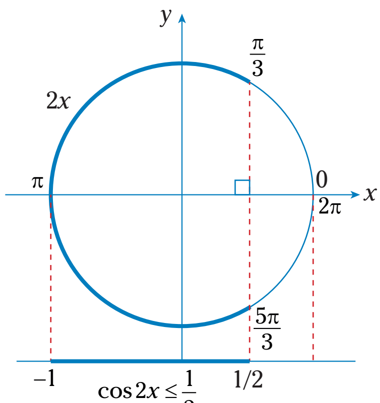
positivo y se verifica si $\cos 2x=-1$

$\rightarrow 2x=\pi;\ 3\pi;\ 5\pi;\ \ldots$

$x=\frac{\pi}{2};\ \frac{3\pi}{2};\ \frac{5\pi}{2}$

cumplen la relación y ninguno es entero

$$\rightarrow 2\cos 2x-1\leq 0\rightarrow\cos 2x\leq\frac{1}{2}$$



De la C.T.

$$2x\in\left[\frac{\pi}{3};\frac{5\pi}{3}\right]\cup\left[\frac{7\pi}{3};\frac{11\pi}{3}\right]\cup\left[\frac{13\pi}{3};\frac{17\pi}{3}\right]$$

$$x\in\left[\frac{\pi}{6};\frac{5\pi}{6}\right]\cup\left[\frac{7\pi}{6};\frac{11\pi}{6}\right]\cup\left[\frac{13\pi}{6};\frac{17\pi}{6}\right]$$

$>2\pi$

0,52 2,61 3,66 5,75

nº de valores enteros $\{1;\ 2\}$ — nº de valores enteros $\{4;\ 5\}$

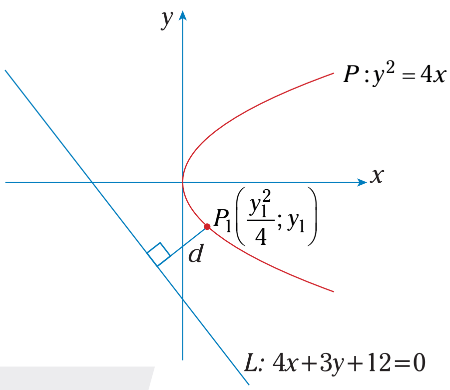
Soluciones enteras: 4

Clave: D

Resolución N.°38

Del enunciado:

d: menor distancia de P a L



Si $P_1\in P$ y allí es la menor distancia si $y=y_1\rightarrow x=\frac{y_1^2}{4}$

Por distancia de punto a recta

$$d=\frac{\left|4\left(\frac{y_1^2}{4}\right)+3(y_1)+12\right|}{\sqrt{4^2+3^2}}$$

$$d=\frac{|y_1^2+3y_1+12|}{5}$$

$$d=\frac{\left|\left(y_1+\frac{3}{2}\right)^2+\frac{39}{4}\right|}{5}$$

$$d_{\text{mínimo}}\text{ si } y_1=-\frac{3}{2}\ \wedge\ d_{\text{mín}}=\frac{39}{20}$$

Clave: B

Resolución N.°39

$M^2+N^2=?$

$M=\tan(\alpha-\beta)_{máx} \wedge N=\tan(\alpha-\beta)_{mín}$

Además $\tan\alpha=3\tan\beta$

$$\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$

Reemplazando dato

$$\tan(\alpha-\beta)=\frac{3\tan\beta-\tan\beta}{1+3\tan\beta\tan\beta}$$

$$\tan(\alpha-\beta)=\frac{2\tan\beta}{1+3\tan^2\beta}$$

$$\tan(\alpha-\beta)=\frac{1}{\frac{3}{2}\tan\beta+\frac{1}{2}\cot\beta}$$

$$\frac{3}{2}\tan\beta+\frac{1}{2}\cot\beta\le-\sqrt{3}\wedge\frac{3}{2}\tan\beta+\frac{1}{2}\cot\beta\ge\sqrt{3}$$

$$\frac{1}{3\tan\beta+\cot\beta}\in\left[-\frac{\sqrt{3}}{3};0\right\rangle\cup\left\langle 0;\frac{\sqrt{3}}{3}\right]$$

pero $\tan(\alpha-\beta)$ puede ser 0

$$\rightarrow\tan(\alpha-\beta)\in\left[-\frac{\sqrt{3}}{3};\frac{\sqrt{3}}{3}\right]$$

Luego $M=\frac{\sqrt{3}}{3} \wedge N=-\frac{\sqrt{3}}{3}$

$$\therefore M^2+N^2=\frac{1}{3}+\frac{1}{3}=\frac{2}{3}$$

Clave: A

Resolución N.°40

$$\cos^2x+2\cos x+\frac{2}{\cos x}+\frac{1}{\cos^2x}=1$$

$$\cos^2x+\frac{1}{\cos^2x}+2\left(\cos x+\frac{1}{\cos x}\right)=1$$

Sea $\cos x+\frac{1}{\cos x}=t\rightarrow\cos^2x+\frac{1}{\cos^2x}=t^2-2$

Reemplazando

$$t^2-2+2t=1\rightarrow t^2+2t+1=3+1$$

$$(t+1)^2=4\rightarrow t+1=\pm2\rightarrow t=1 \vee t=-3$$

Como $\cos x+\frac{1}{\cos x}\le-2 \vee \ge2\rightarrow\cos x+\frac{1}{\cos x}=-3$

$$\cos^2x+3\cos x+1=0$$

$$\cos^2x+3\cos x+\frac{9}{4}=-1+\frac{9}{4}$$

$$\left(\cos x+\frac{3}{2}\right)^2=\frac{5}{4}$$

$$\cos x=\underset{\text{(SI)}}{\frac{\sqrt{5}-3}{2}} \vee \cos x=\underset{\text{(NO)}}{\frac{-\sqrt{5}-3}{2}}$$

$$x=2n\pi\pm\arccos\left(\frac{\sqrt{5}-3}{2}\right),\ n\in\mathbb{Z}$$

$\therefore$ menor solución positiva $\arccos\left(\frac{\sqrt{5}-3}{2}\right)$

Clave: A